Nonlocal solitons and filamentation in soft matter

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People and references

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References:

Phys Rev Lett 97, 123903 (2006), arXiv:cond-mat/0608425

Phys Rev Lett 95, 183902 (2005), arXiv:physics/0510172

Synopsis

- Soft matter
- Optical nonlinearities in soft matter
- Electrostriction
- A novel model of electrostrictive nonlinearity
- The static structure factor
- Paraxial and non paraxial solitons
- Modulational instability and filamentation
- Experiments from the literature
- Conclusions and perspectives

Soft Matter

Soft matter

Recent review: C. N. Likos, Phys. Rep. 348, 267 (2001)

- Liquid crystals
- Soft-like materials
  - Special classes of photorefractive crystals and glasses
- Colloids
  - Include foams, polymers in liquids, glues, blood, biomatter ....

Optical nonlinearities in soft-matter

Optical nonlinearities in soft-matter

Time scales and relative strengths strongly dependent on the specific soft-material

- Re-orientational
  - negligible for isotropic particles
- Thermal
  - negligible in the absence of absorption
- Thermophoresis (Ludwig-Soret effect)
  - thermally driven diffusion
  - negligible in the absence of absorption
  - see our experimental paper: cond-mat/0609659
- Electrostriction
  - optical tweezing and all of that…
  - always present !
  - let's start from this

Electrostriction

- Palmer, Ashkin and others, '80s
- Particles go in the region with the highest optical intensities
- Always focusing
- Previous model: standard Kerr nonlinearity
- NO 2D SOLITONS !!!!

$$n = n_0 + n_2 I$$

$$n_2 = \frac{4\pi^2 r_s^6 n_h^4}{c n_0^2} \left(\frac{\epsilon_s - \epsilon_h}{\epsilon_s + 2\epsilon_h} \right)^2 \frac{\rho_0 S_0}{k_B T}$$

n_2 is given in terms of refractive index, temperature, compressibility…

Electrostriction in the presence of inter-particle correlation

- Previous theory neglects particle-particle interaction

- We want to include the particle-particle interaction in order to determine the role of material structure in the nonlinear optical response

Ab-initio approach....

- Particles evolve according to standard molecular dynamics

$$\ddot{r}_j + 2\gamma\, \dot{r}_j = -\partial_{r_j} V + F_{el}$$

V: interparticle potential
F_{el}: electrostrive force

- The optical field evolves according to Maxwell-equations in a disordered colloidal material

TOO MUCH COMPLICATED!

(see also Conti, Angelani, Ruocco, cond-mat/0607534)

The mode-coupling theory approach

[density-]mode-coupling theory (MCT) for soft-matter

- Bengtzelius, Gotze, Sjolander, 1984
- From the inter-particle interaction potential an equation is derived for the density of the particles
- This approach has been experimentally verified in various "soft" and "glassy" systems
- The MCT approach is also valid in out-of-equilibrium dynamics

$V(r_i - r_j)$ Interparticle potential

↓

MCT equations
(in spatial Fourier coords q=wave-vector)

$$\ddot{\tilde{\rho}}(q,t) + q^2 \frac{k_B T}{S(q)} \tilde{\rho} + q^2 \int_0^t m(t-t') \dot{\tilde{\rho}}(q,t') dt' = \tilde{f}(q,t) + \frac{1}{2} \gamma_e \eta q^2 \tilde{I}(q),$$

Density Fluctuactions term

We added this electrostrive term

Alternative approaches

- Equilibrium Complex fluids theory
- Generalized thermodynamics

Provide the same result ! % at equilibrium

See Phys Rev Lett 95, 183902 (2005), arXiv:physics/0510172

The non-local electrostriction theory

The nonlocal electrostriction theory

$$\Delta n = \rho(\partial n/\partial \rho)_{\rho_0}$$

Refractive index variation

$$\tilde{\rho}(q) = \frac{\gamma_e \eta}{2} \frac{S(q)}{k_B T} \tilde{I}(q)$$

Density variation

It needs:

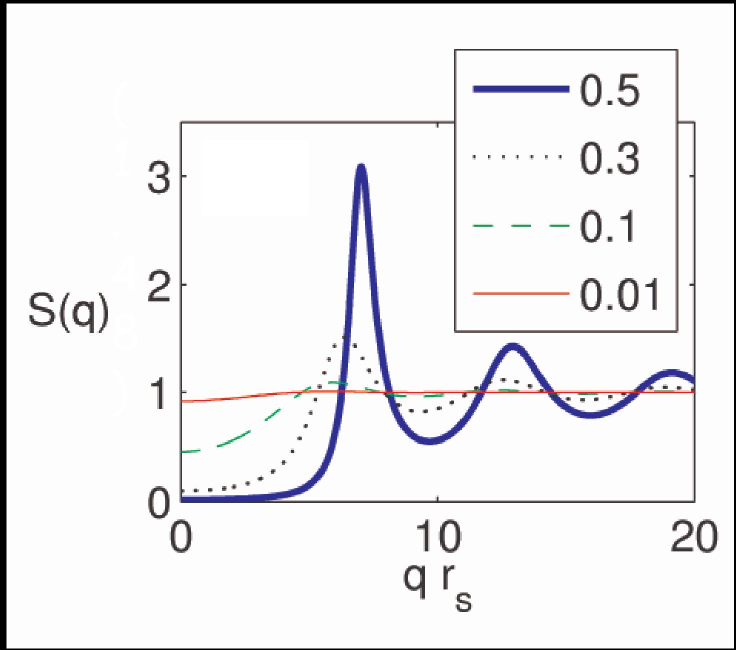
- the static structure factor (i.e. the interparticle potential)
- material polarization parameters (refractive index etc...)

The static-structure factor S(q)

- Hard spheres
- Fractal aggregates

Hard spheres

- V(r)=infinity for r<a
  V(r)=0 for r>a
- Very-short range particle interaction
- While increasing particle packing the non-locality increases…
- … but the strength of nonlinearity is reduced
- For typical experiments in optics non-locality is negligible in hard sphere systems

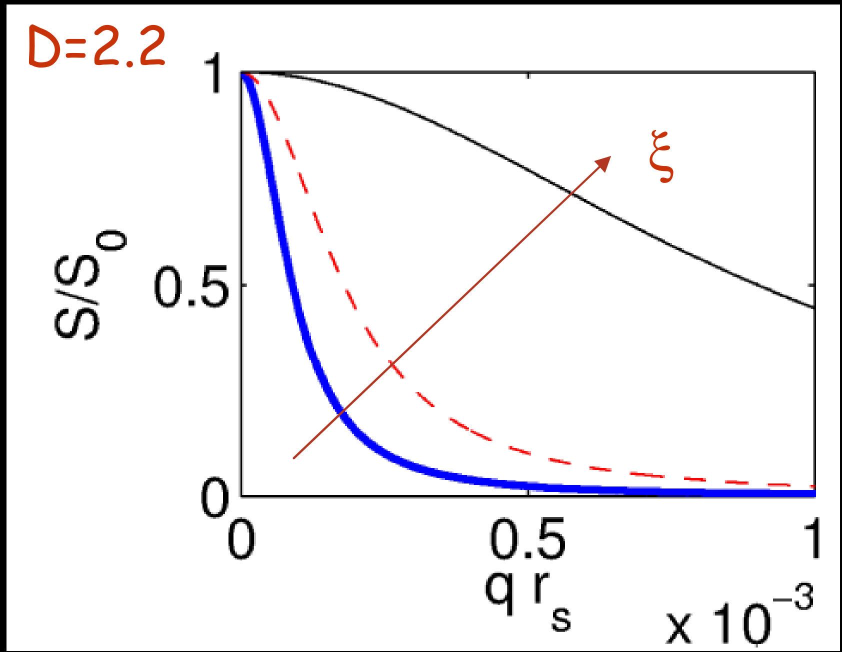


S(q) for increasing packing fraction

r_s : sphere dimension

Fractal aggregates

- Various interaction potentials lead to a universal "shape" for S(q)
- Parametrized by
  - Fractal dimension D
  - Aggregate dimension ξ
- Non-locality is not negligible!



S(q) for increasing aggregate dimension

r_s : sphere radius

Unidirectional equations for the field

- See Kolesik, Wright, Moloney, Phys. Rev. Lett. 92, 253901 (2004)
- Involving only forward propagating beams Maxwell equations for a monochromatic beam can be simplified
- Non-paraxial terms can be treated self-consistently

$$i\frac{\partial \mathcal{E}}{\partial z} + \sqrt{k^2 + \nabla_\perp^2}\,\mathcal{E} + \frac{\omega}{2cn_0}P_{nl}(\mathcal{E}) = 0,$$

$$P_{nl}(\mathcal{E}) = \Delta\chi\mathcal{E} = 2n_0\Delta n\mathcal{E}$$

$$\Delta n = \rho(\partial n/\partial \rho)_{\rho_0}$$

Paraxial approximation and radial symmetry

$$2ik\frac{\partial E}{\partial z} + \nabla_\perp^2 E + \frac{2k^2}{n_0}\left(\frac{\partial n}{\partial \rho}\right)_{\rho_0} \rho E = 0.$$

"Standard nonlocal model"

$$i2k\frac{\partial E}{\partial z} + \nabla_\perp^2 E + \chi E \int_0^\infty G(r, r')|E(r', z)|^2 r' dr' = 0,$$

Radial symmetry

$$G(r, r') \equiv \int_0^\infty \frac{S(Q)}{S_0} J_0(Qr) J_0(Qr') Q dQ,$$

The non locality kernel is given the S(q)

Nonlocal solitons in soft-matter

The leading equations % adimensional

- Equations are written in adimensional form
- Bound states are found with wave-vector β
- Are solved by an iterative Newton-Rampson technique
- The non-paraxiality is measured by ε

$$\varepsilon = (\lambda/4\pi n w_0)^2$$

$$\nabla^2_\sigma u - \beta u + u \int_0^\infty g(\sigma,\sigma')u^2(\sigma')\sigma' d\sigma' = 0,$$

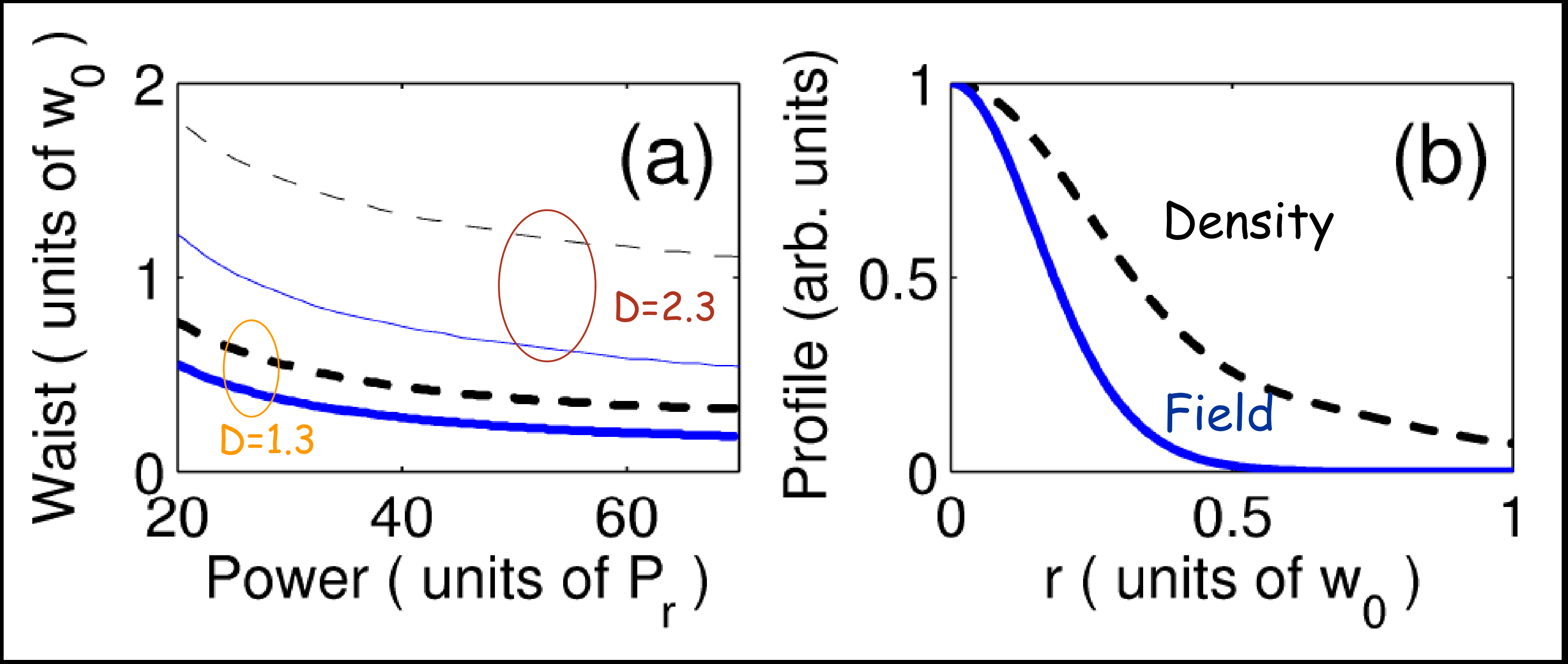
Paraxial nonlinear nonlocal equation

$$\nabla^2_\sigma u - \varepsilon \nabla^4_\sigma u - \beta u + u \int_0^\infty g(\sigma,\sigma')u^2(\sigma')\sigma' d\sigma' = 0,$$

Non-paraxial nonlinear nonlocal equation

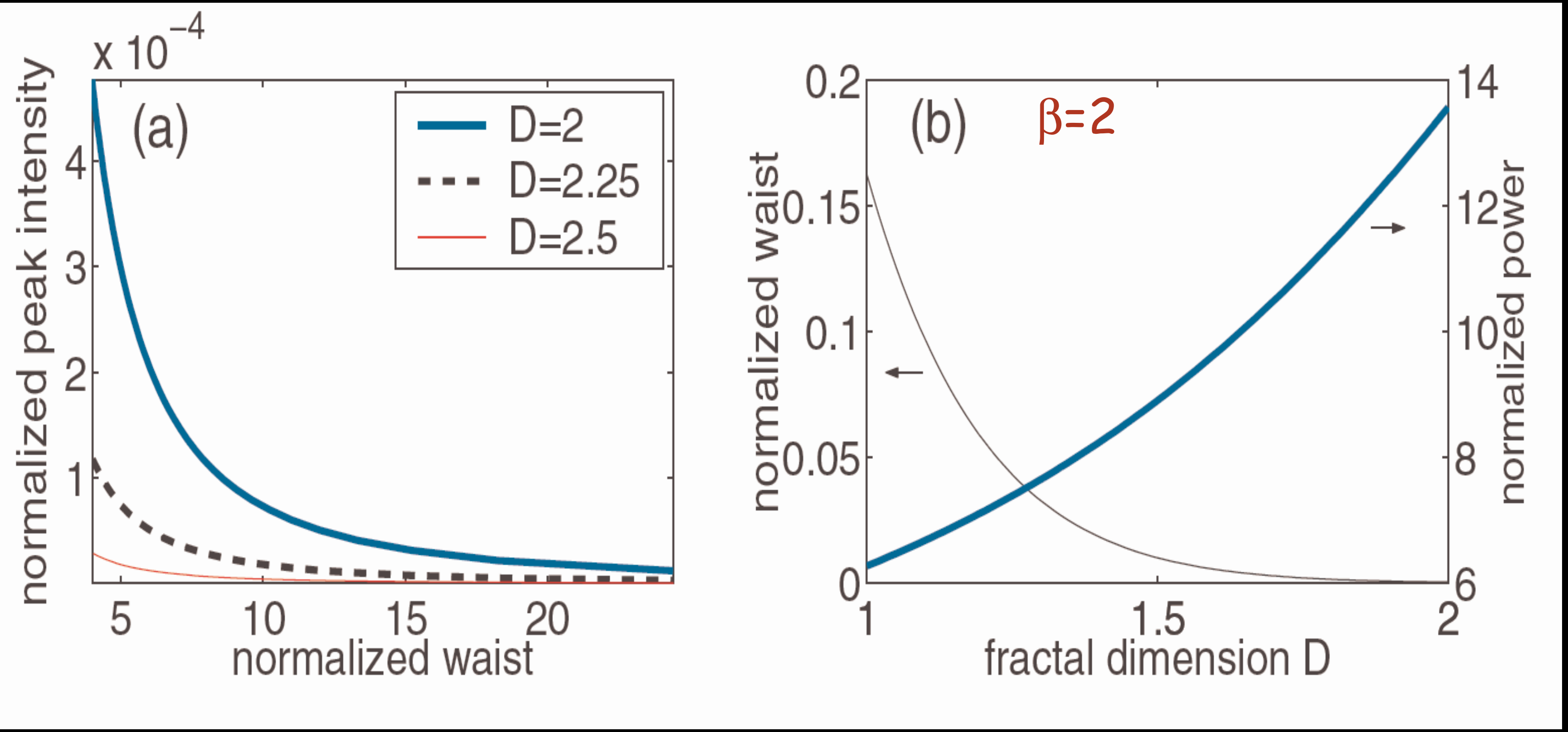
Soliton solutions (paraxial)

Existence curve and soliton profiles for different fractal dimensions



Dashed line: density perturbation

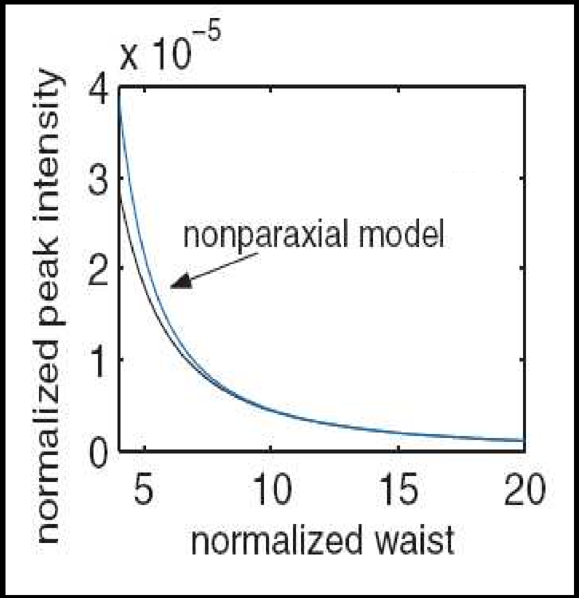
Soliton features versus fractal dimension



Existence curve for different D

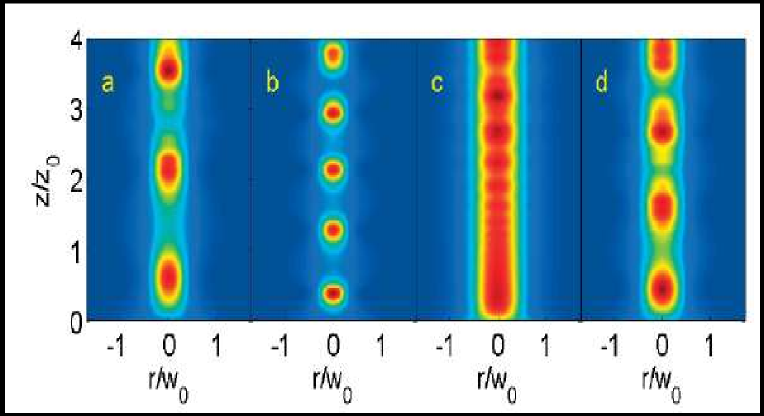
Waist and power Vs D for fixed β

Paraxial Vs non-paraxial



Correction to the existence curve due non-paraxial effects (D=2.5)

ε=0.0 ε=0.01



Stable propagation for paraxial and non-paraxial models (D=2.5)

a,b : different powers, paraxial

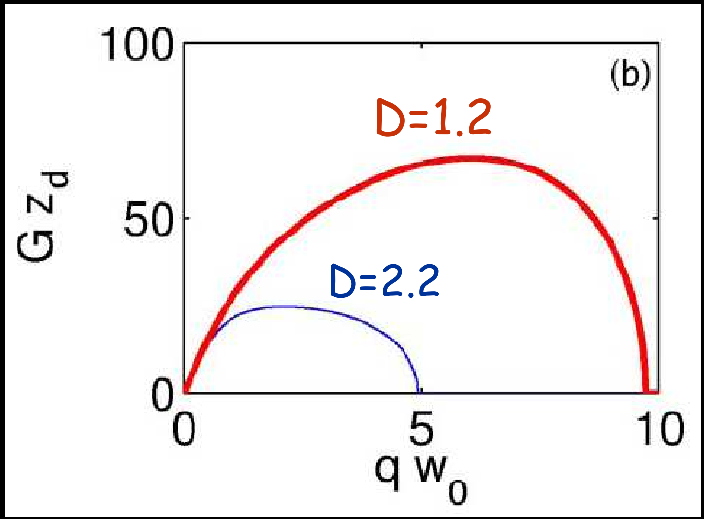
c,d : different powers, non-paraxial

Noise and Modulational Instability

- MCT implies fluctuactions
- Noise
- Stochastic PDE
- The MI gain is affected by the fractal dimension

$$i\frac{\partial a}{\partial \zeta} + \frac{1}{2}\nabla^2_{\mathrm{st}} a + \psi a = 0; \qquad \tilde{\psi} = R(q_s, q_t)(\widetilde{|a|^2} + \tilde{\nu}),$$

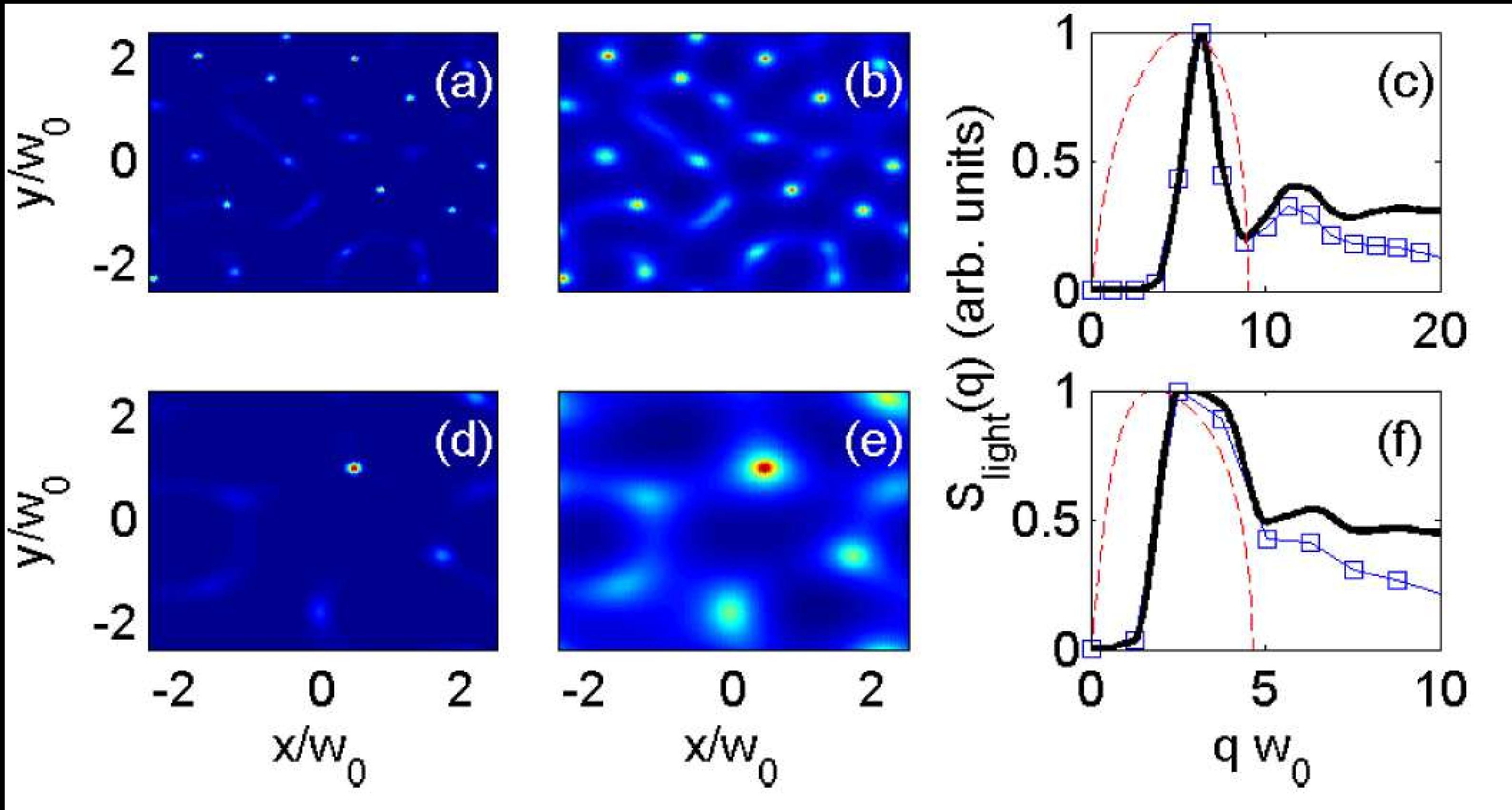
Stochastic nonlocal PDEs



MI gain profile for two different
fractal dimensions D

MI and filaments % just paraxial

Intensity | Density | Light distribution averaged over 40 noise realizations



D=1.3

D=2.3

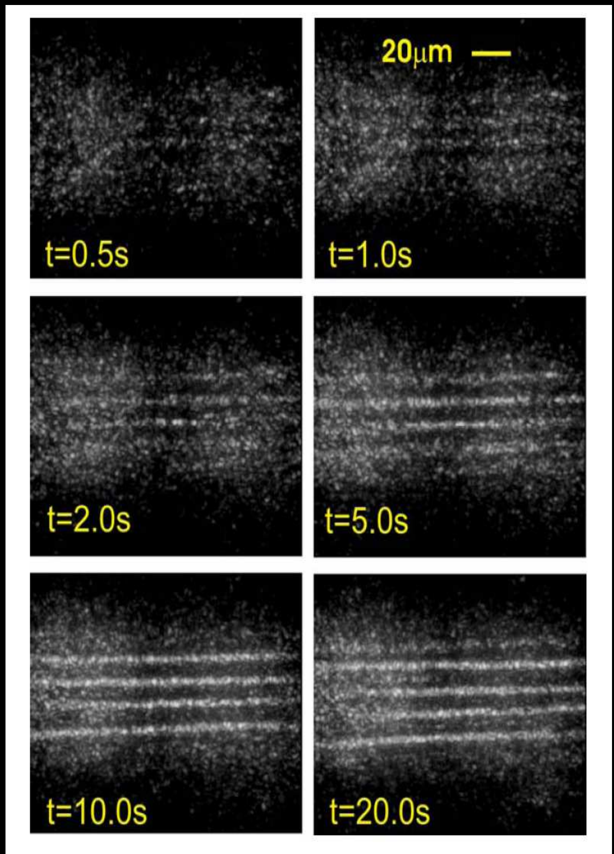
Recent experiments by an independent group …

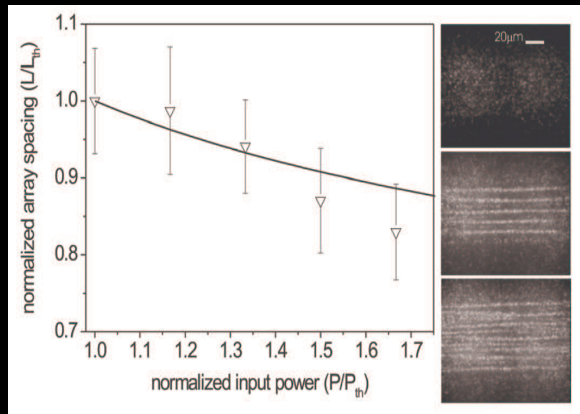
physics/0701252

Reece, Wright, Dholakia

Experiments in St Andrews

physics/0701252 Reece, Wright, Dholakia





- 1D geometry
- Formation of an array of solitons
- Colloidal suspension (hard spheres?) , radius 420 nm

Other experiments in V. E. Yashin and others, Opt. Spectrosc. **98** (3), 466-469 (2005).

Conclusions and developments

- Conclusions % what we did
  - A nonlocal theory for electrostriction in soft-matter
  - Ultra-thin solitons and MI in soft-matter
  - The role of the fractal dimension in optical solitons and MI

- Developments % what we would like to do
  - Observation of 2D non-local solitons in soft-matter
  - Other S(q), including thermal and re-orientational
  - Time-dynamics
  - Applications to bio-physics and to optical tweezing

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