Complexity in Random Lasers

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Nice Days of Waves in Complex Media, 7-9 June 2010

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Synopsis

- **Part I (COMPLEXITY)**
  - Theoretical model
  - Complexity
  - Phase diagram
- **Part II (Haus equation)**
  - The Master Equation for Random Lasers
- **Part III (FDTD, not reported)**
  - FDTD of 3D Anderson localization of light
  - FDTD of 1D and 3D random lasers

COMPLEXITY AND SPIN GLASS THEORY

Spin Glass theory

- Early experimental works in 1959-1960
- Driving theoretical results in 1970-1980
- Extension to a broad range of physical settings till today
  ("Science of complex systems")

  - Soft matter and structural glasses
  - Optimization and neural networks
  - Brain modeling
  - Econophysics
  - …
  - Random lasers (and nonlinear optics) ?

Mezard, Parisi and Virasoro, *Spin Glass Theory and Beyond* (not for beginners !)

Castellani and Cavagna, *Spin-Glass Theory for Pedestrians,* (not for pedestrian !)
arXiv:cond-mat/0505032

Spin glasses

- Hamiltonian
- Quenched disorder

$$H = H(\sigma; J)$$

σ dynamic variables (spins)
J random couplings (or quenched variables)

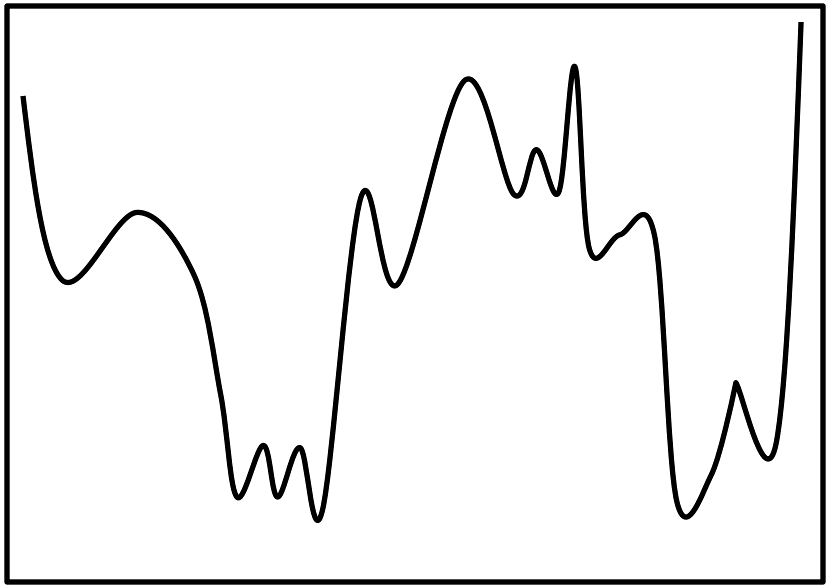
- Examples:
  - Mn in Cu
  - Shakespeare
  - Biological systems

$$H = -\sum_{<ij>} J_{ij}\sigma_i\sigma_j$$

Spin glass theory and Complexity

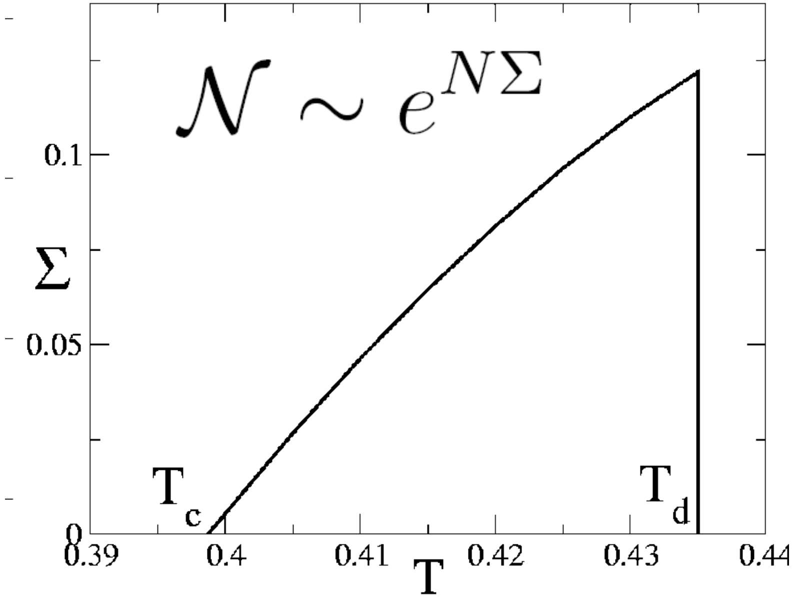
- An exponentially large number of states $\mathcal{N}$
- Replica Symmetry Breaking (or "glassy") transitions

"Energy landscape"



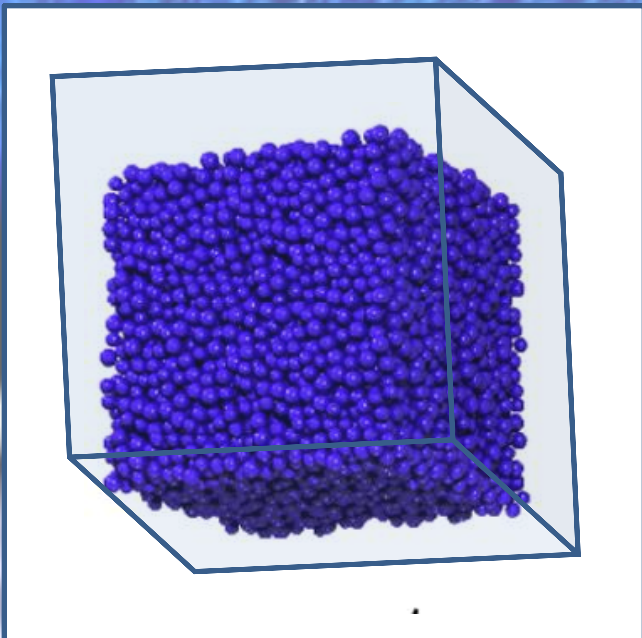
Σ: "complexity"

or "configurational entropy"

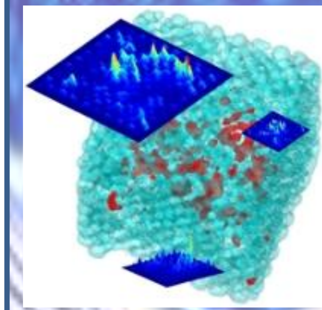


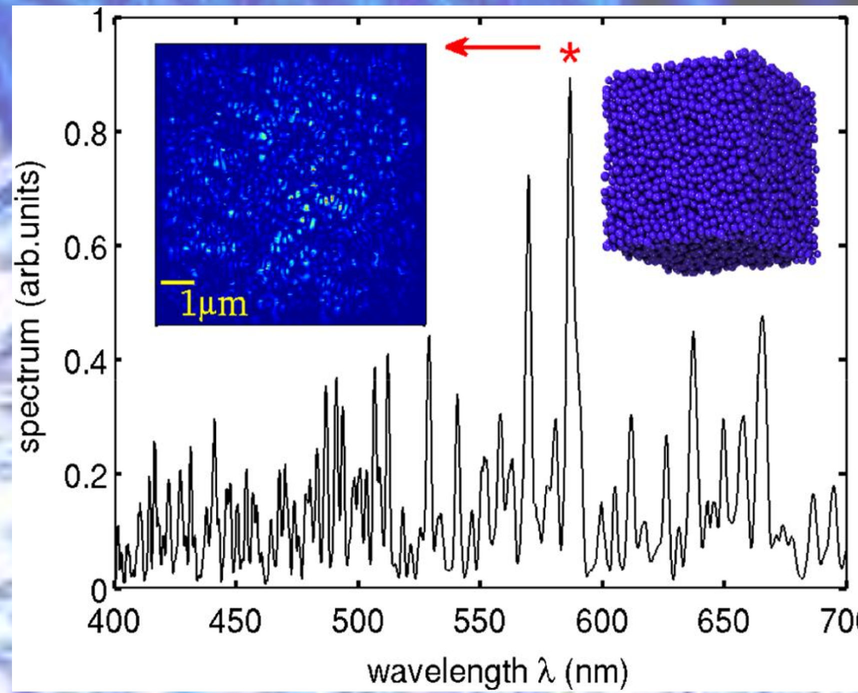
- Ergodicity breaking

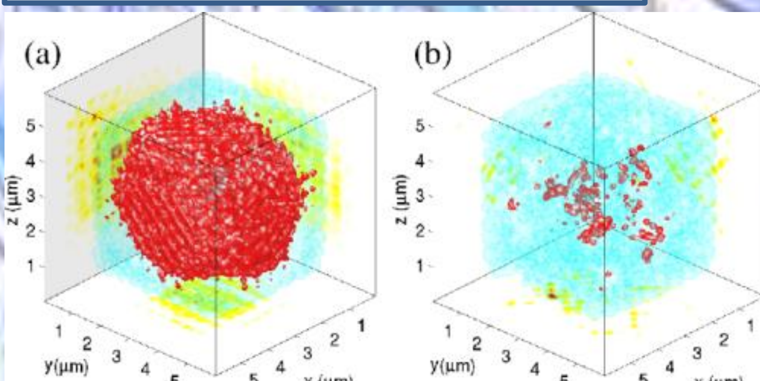
Disordered electromagnetic cavity



- A random distribution of the refractive index
- Closed Cavity (the modes are complete set)
- Open Cavity (modes are linearly coupled)







Conti et al. Nature Physic s 2008

3D+1 FDTD OF RL, Conti, PRL 2008

Electromagnetic Modes

Modes are used to express the general field

$$\mathbf{E} = \mathrm{Re}\left[\sum_n \sqrt{\omega_n} a_n(t) \mathbf{E}_n(\mathbf{r}) \exp(-i\omega_n t)\right]$$

$$\omega_m |a_m|^2$$

Electromagnetic energy

Electromagnetic energy

$$\mathcal{E}_{EM} = \int \mathbf{E} \cdot \mathbf{D} dV$$

Instantaneous energy

$$\mathbf{D} = \epsilon_0 \epsilon_r(\mathbf{r})\mathbf{E} + \epsilon_0 \mathbf{P}_{NL}$$

Linear and nonlinear polarization

$$\mathbf{E} = \mathrm{Re}\left[\sum_n \sqrt{\omega_n} a_n(t) \mathbf{E}_n(\mathbf{r}) \exp(-i\omega_n t)\right]$$

Mode expansion

Time averaged interaction Hamiltonian

$$H = \langle \int \epsilon_0 \mathbf{E} \cdot \mathbf{P}_{NL} dV \rangle = \sum_{\omega_s+\omega_p=\omega_q+\omega_r} g_{spqr} a_s a_p a_q^* a_r^*$$

Coupling coefficients

$$g_{spqr} = \frac{\sqrt{\omega_s \omega_p \omega_q \omega_r}}{2i} \int_V \chi_{\alpha\beta\gamma\delta}(\omega_s; \omega_q, \omega_r, -\omega_p, \mathbf{r}) \times E_s^{\alpha}(\mathbf{r}) E_p^{\beta}(\mathbf{r}) E_q^{\gamma}(\mathbf{r}) E_r^{\delta}(\mathbf{r}) dV.$$

Random coefficients

The quenched amplitude regime

Interaction H

$$H = \langle \int \epsilon_0 \mathbf{E} \cdot \mathbf{P}_{NL} dV \rangle = \sum_{\omega_s + \omega_p = \omega_q + \omega_r} g_{spqr} a_s a_p a_q^* a_r^*$$

The mode amplitudes are taken quenched variables

$$a_m(t) = A_m(t) \exp[i\varphi_m(t)]$$

$$\mathcal{H}(G,\varphi) = \mathcal{H}_o + \sum_{\{sp\},\{qr\}} G_{spqr} \cos(\varphi_s + \varphi_p - \varphi_q - \varphi_r)$$

g : mode overlaps

$$G_{spqr} = g_{spqr} A_s A_p A_q A_r$$

Gaussian variables

A mode amplitudes

Normalized Hamiltonian

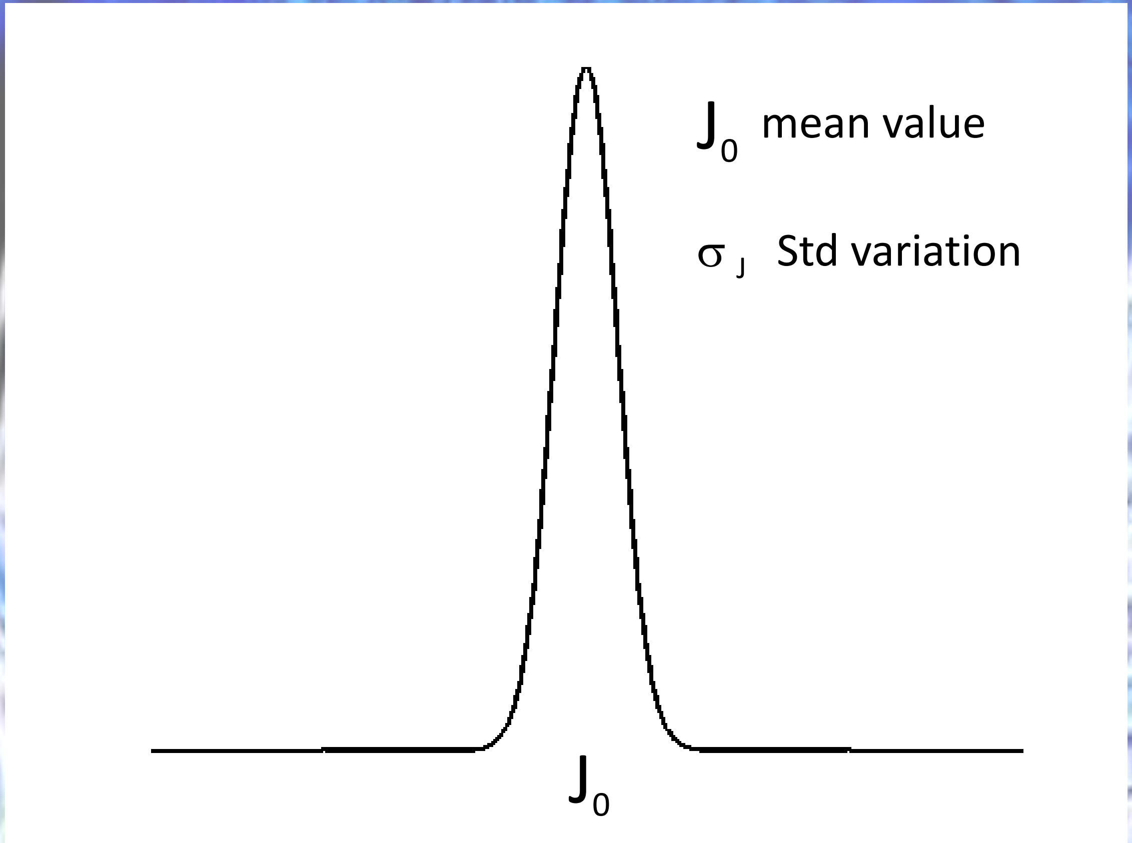
$$\mathcal{H}(G,\varphi) = \mathcal{H}_o + \sum_{\{sp\},\{qr\}} G_{spqr} \cos(\varphi_s + \varphi_p - \varphi_q - \varphi_r)$$

$$H = \sum_{\{pr\},\{sp\}} J_{spqr} \cos(\varphi_s + \varphi_p - \varphi_q - \varphi_r)$$

PRL 96, 065702 (2006); PRB 74, 104207 (2006); PRB 76, 064202 (2007); PRL 102, 083901 (2009)

The coupling coefficients

Distribution of the couplings (three cases)



degree of order J_0/σ_J

Ordered laser

$\sigma_J=0$

$J_0/\sigma_J \to \infty$

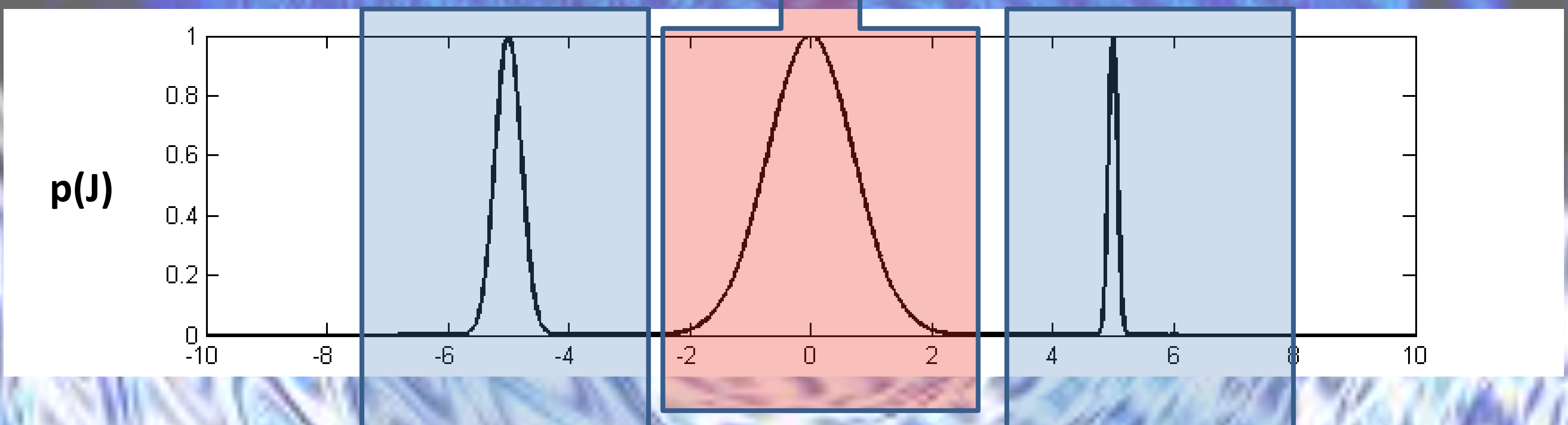
Disordered laser

$\sigma_J \to \infty$

$J_0/\sigma_J \to 0$

Three examples

Completely disordered laser (J_0=0)



Weakly Ordered laser (J_0<0)

Ordered laser (J_0>0)

Spin-glass approach

Start from a dimensionless Hamiltonian
and partition function

$$H = -\sum_{\{i\},\{k\}} J_{i_1,i_2,k_1,k_2} \cos(\varphi_{i_1} + \varphi_{i_2} - \varphi_{k_1} - \varphi_{k_2})$$

$$Z(J) = \int d\{\varphi\} e^{-\beta H(J,\varphi)}$$

Calculate the partition function
- use the replica "trick"
- average over disorder
- some manipulations
- thermodynamic limit (N->Inf)

$$-\beta f = \lim_{N\to\infty} \frac{1}{N} \overline{\ln Z(J)} = \lim_{N\to\infty} \lim_{n\to 0} \frac{\overline{Z^n(J)} - 1}{nN}$$

Identify the "overlaps"

$$q_{ab} = \langle e^{i(\varphi_a - \varphi_b)} \rangle$$

Calculate the transition temperature

$$q = \frac{\int_0^\infty \mathcal{D}z I_0^m(\beta\lambda z) \left[\frac{I_1(\beta\lambda z)}{I_0(\beta\lambda z)}\right]^2}{\int_0^\infty \mathcal{D}z I_0^m(\beta\lambda z)}$$

The order parameters

$$\tilde{m} = \langle\langle e^{\imath\phi}\rangle_{\mathcal{L}}\rangle_{x,\zeta}$$

Magnetizations

The magnetization is different from zero when all the modes are in phase

$$q_{ab} = \langle e^{i(\varphi_a - \varphi_b)}\rangle$$

Overlaps

The overlaps are different from zero when two replicas are trapped in different minima

Thermodynamic phases (qualitatively)

Inchoerent regime (q=0, m=0)

All the modes oscillates independently

Paramagnetic-like

Mode-locking regime (q≥0, m>0)

All the modes oscillate with the same phase

Ferromagnetic like

Need of a saturable absorber

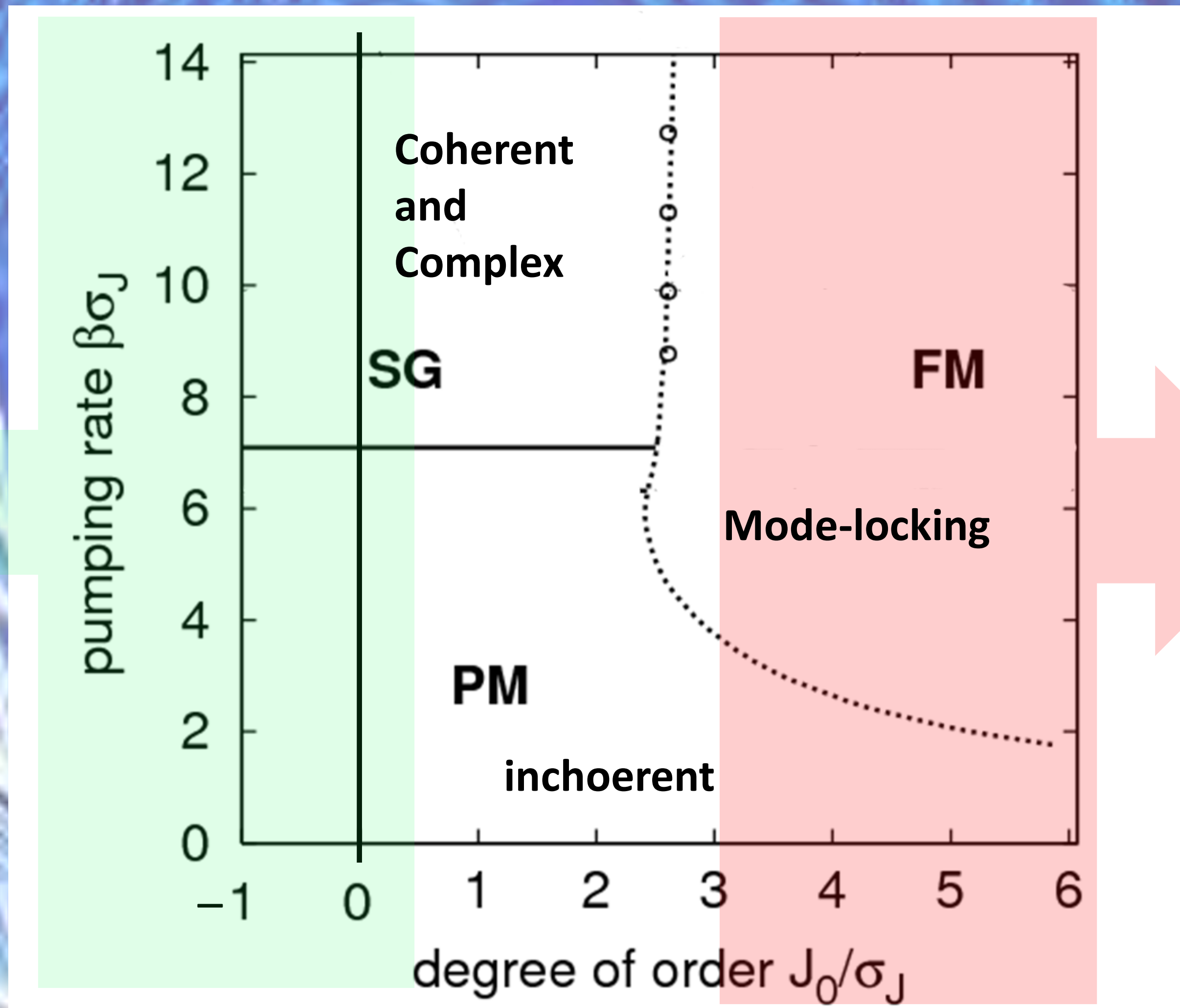
Glassy phase-locking (q>0, m=0)

All the modes oscillate with fixed phase (with zero mean)

Glassy like (“GLASSY LIGHT”)

Universal phase diagram for multi-mode lasers (ordered and disorder)

Random Lasers



Standard lasers

PRL 96, 065702 (2006); PRB 74, 104207 (2006); PRB 76, 064202 (2007); PRL 102, 083901 (2009)

Summarizing

- Multimode ordered and disordered lasers can be treated in a unified way
- Disorder allows coherent phases (!)
- RL have a glassy phase-diagram
- Complexity>0 implies:
  - Large shot to shot spectral fluctuactions
  - Speckle pattern fluctuactions
  - This is dependent on the pump level
- Complexity is much better than CHAOS !

HAUS/GROSS PITAEVSKII EQUATION

The Haus equation

Describes passive mode-locking in multi-mode ordered lasers (ultrafast oscillators)

GAIN = LOSS

$$g_0 a(\tau) + \frac{g_0}{\omega_g^2}\frac{\partial^2 a(\tau)}{\partial \tau^2} + \gamma |a(\tau)|^2 a(\tau) = \alpha_0 a(\tau)$$

Linear gain

finite gain bandwidth

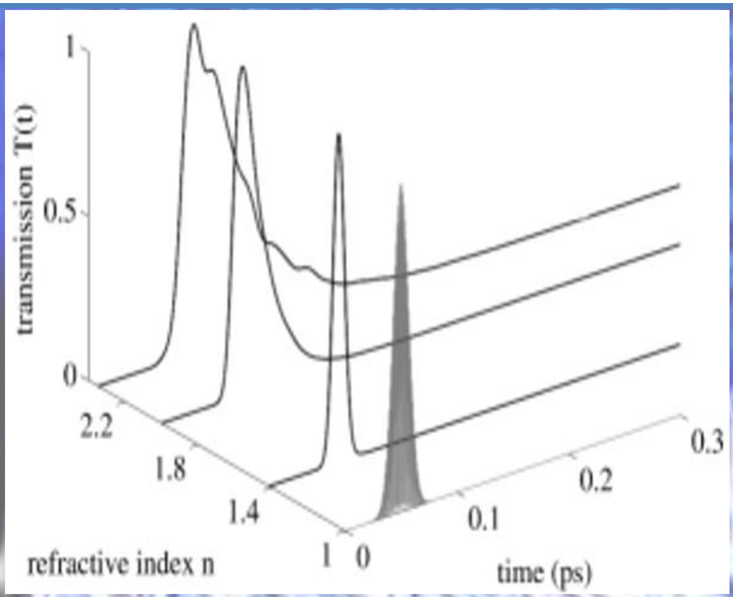
nonlinear part

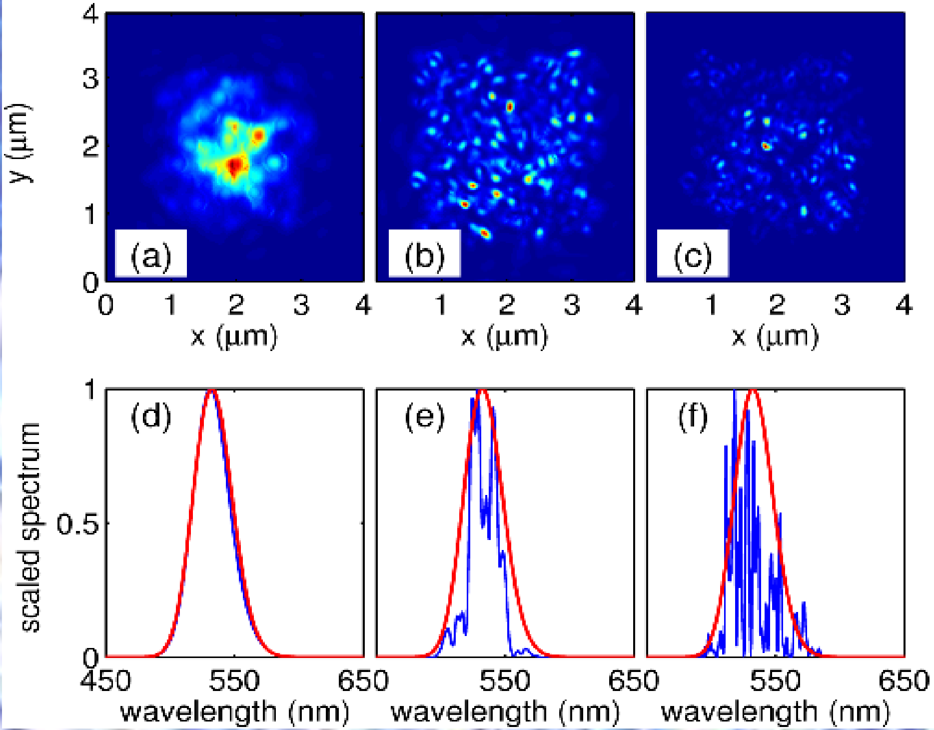
Linear losses

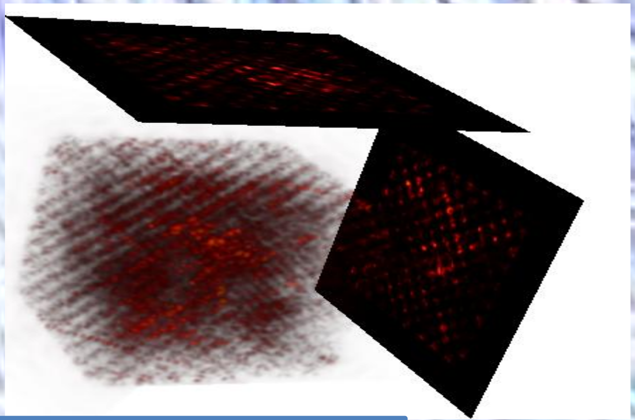
Admits bell-shaped solutions (pulsed emission – broad band) only for positive nonlinearity (need of a saturable absorber)

Mode locking only for $\gamma>0$

Numerical simulations of the Anderson localization of light (3D+1)

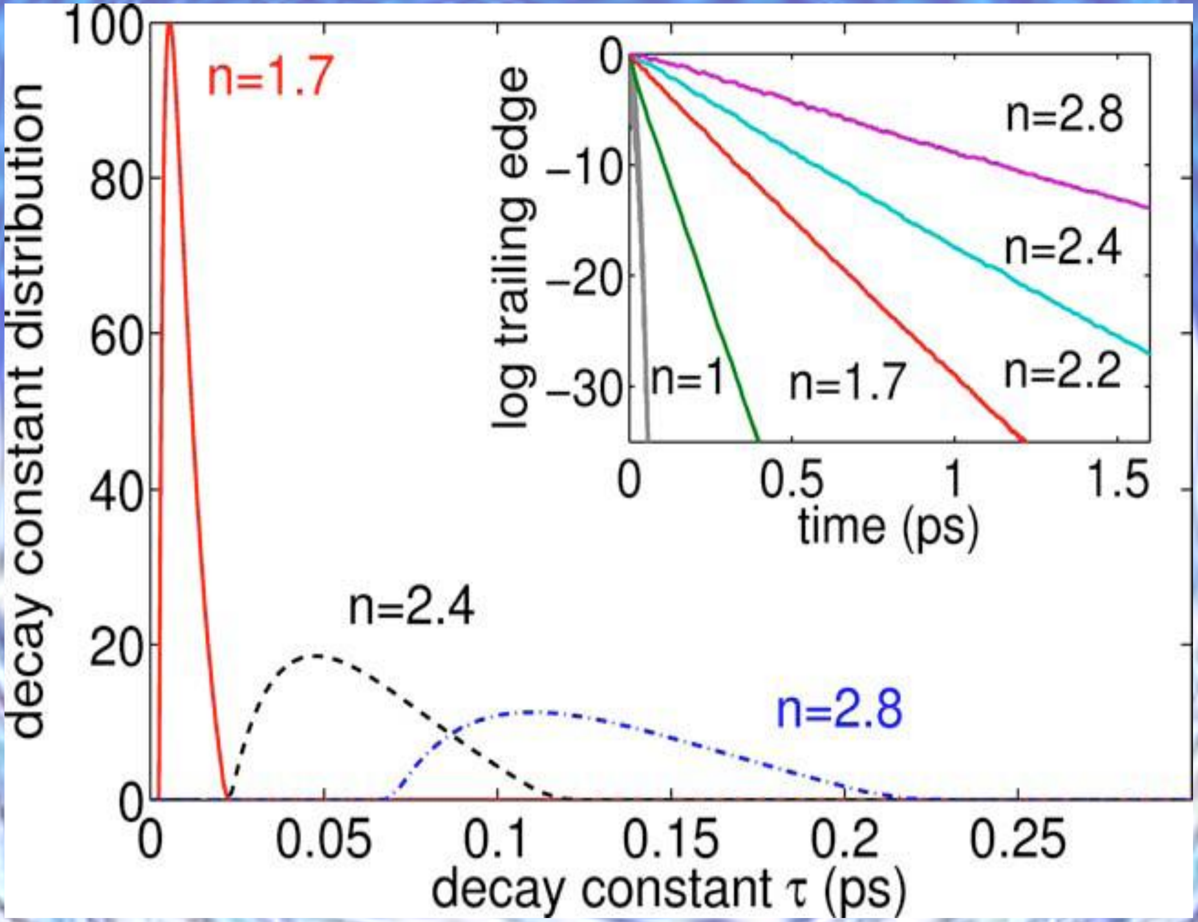




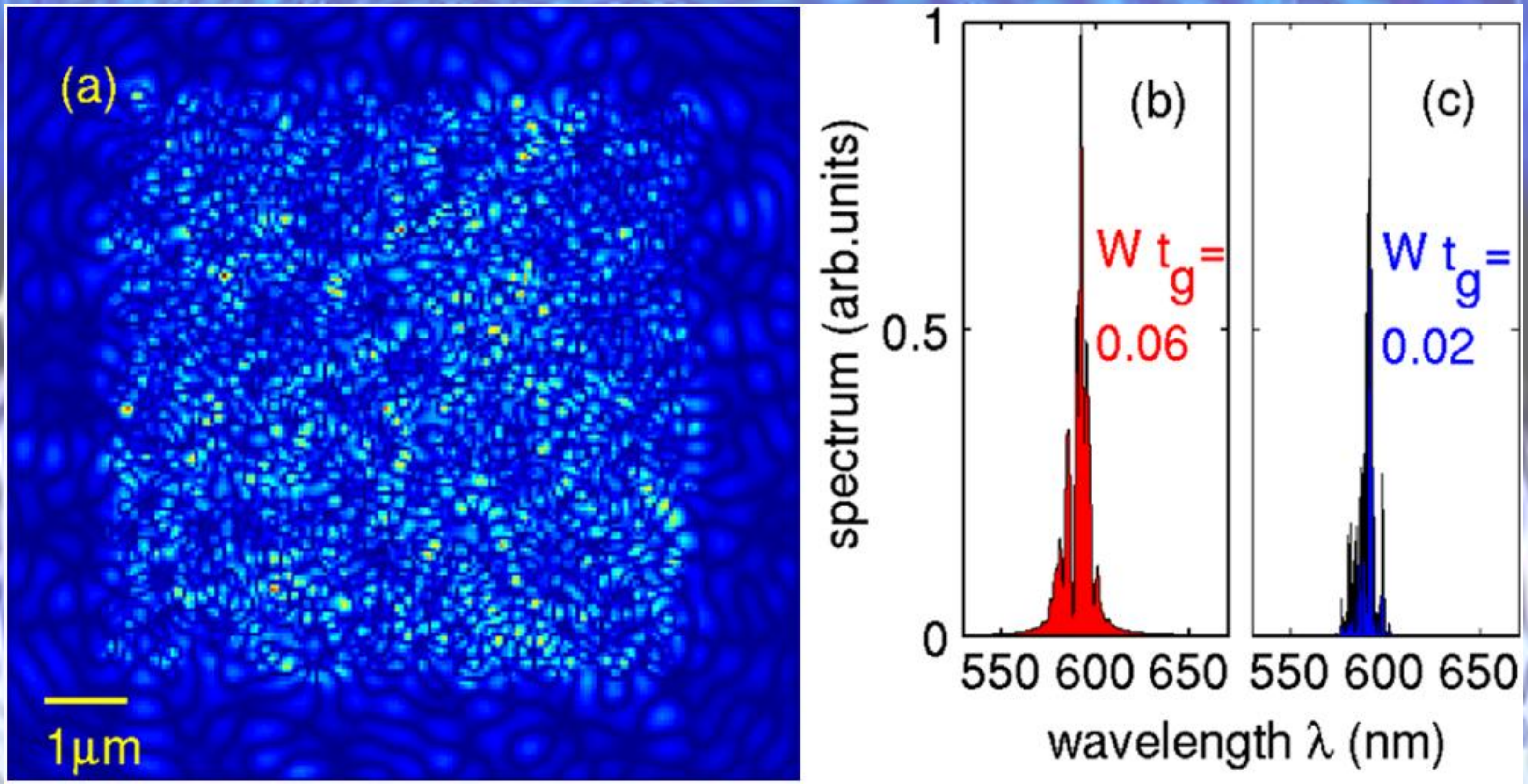


Opt. Lett. 2009, PRB 2010

Decay time distribution after FDTD



Laser emission (Maxwell-Bloch FDTD 3D+1)



PRL 101, 143901 (2009)

Very simple … indeed

The lineshape is described by A(ω)

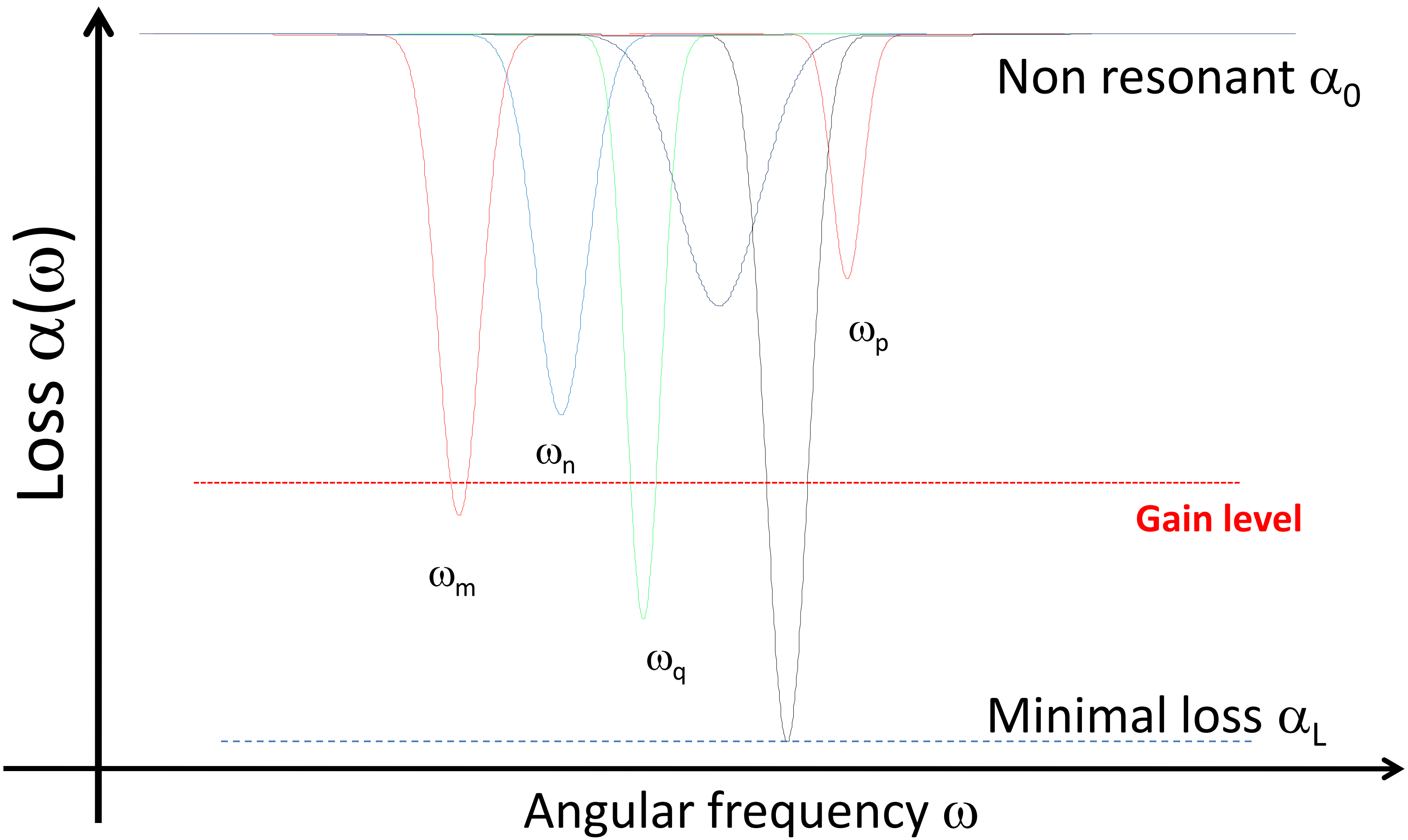
GAIN=LOSS

$$g[A(\omega)] = \alpha_0 A(\omega) - \int \alpha_{\text{avg}}(\omega - \Omega) A(\Omega) d\Omega$$

Standard losses

Linearly coupled resonances

Loss profile



In the time domain …

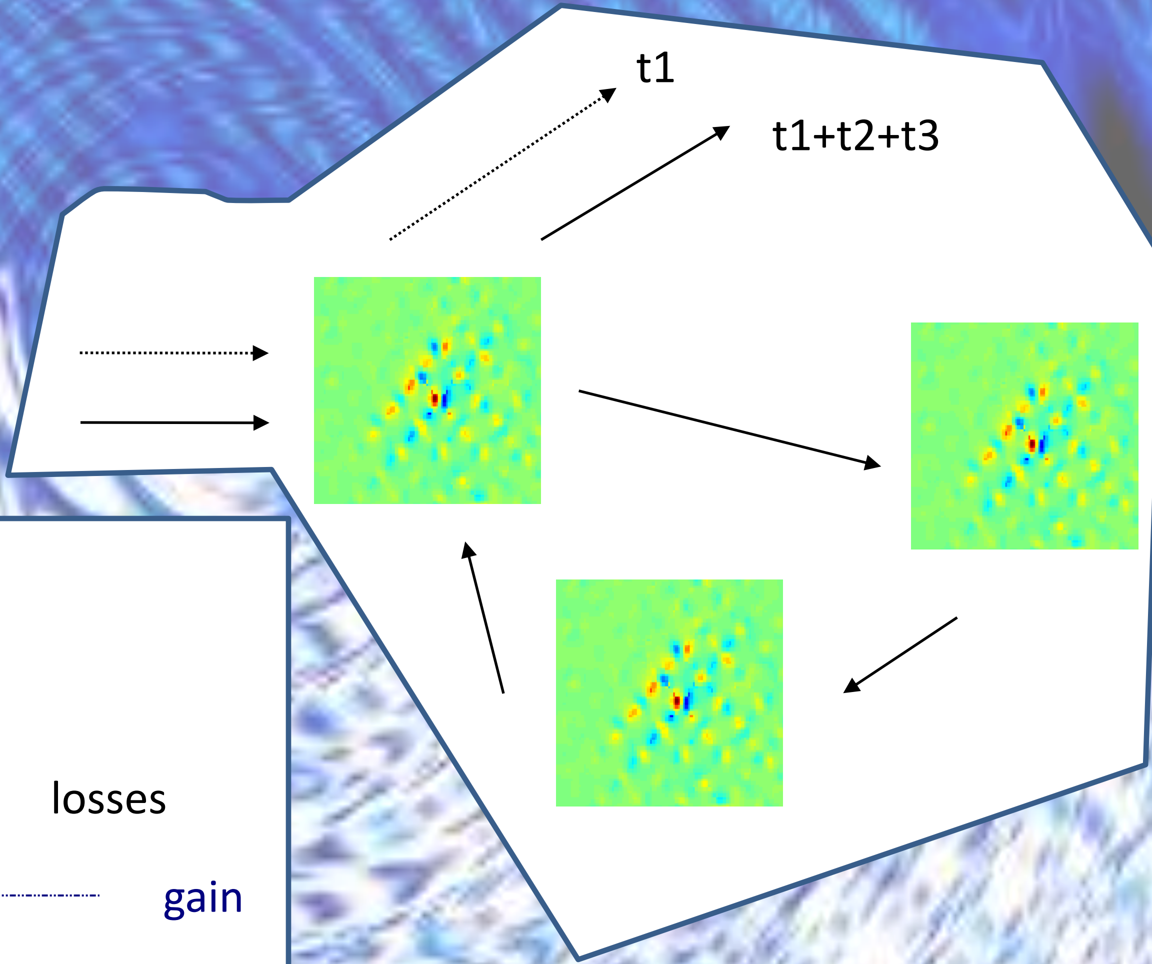
$$g_0\left[a(t) + t_g^2 \frac{d^2 a}{dt^2} - \gamma_s |a|^2 a \right] = [\alpha_0 - \phi_L(t)]a,$$

Here : finite gain bandwidth and gain saturation

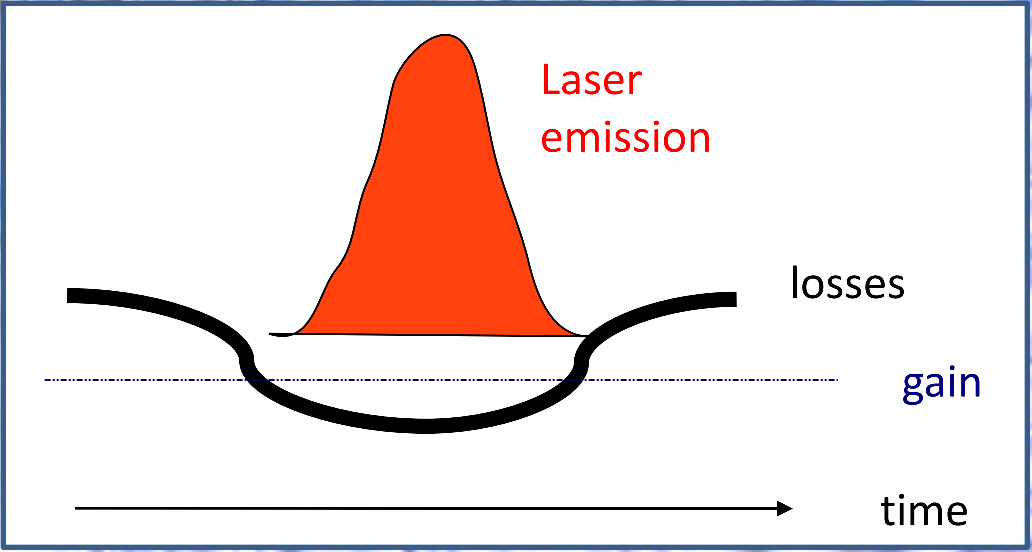
Here: the lifetime distribution

Heuristic interpretation

Photons that are coupled
in various modes
experience a longer lifetime



Pulsed emission



After normalization

$$-\frac{d^2\varphi}{d\tau^2}+\tau^2\varphi+|\varphi|^2\varphi=E\varphi$$

The Haus/Gross-Pitaevskii equation for random lasers

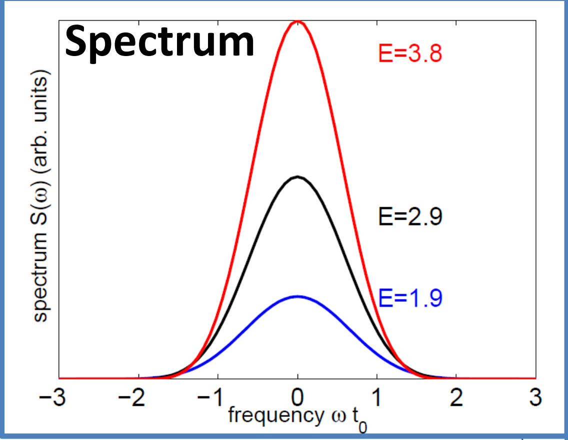
$$E=\frac{t_L}{t_g}\frac{g_0-\alpha_L}{\sqrt{(\alpha_0-\alpha_L)g_0}}=\frac{p-1}{\kappa\sqrt{p}}$$

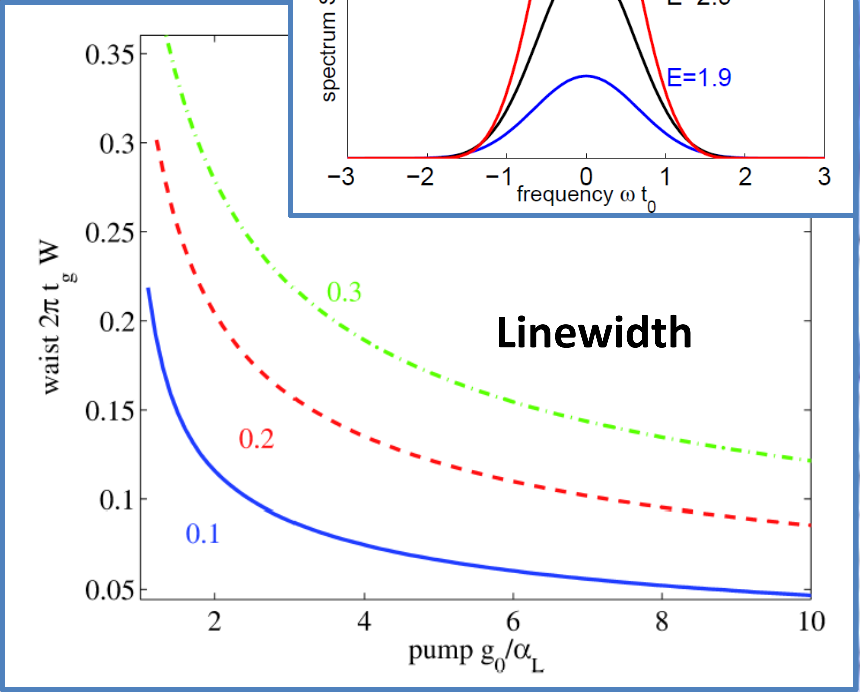
Define a threshold

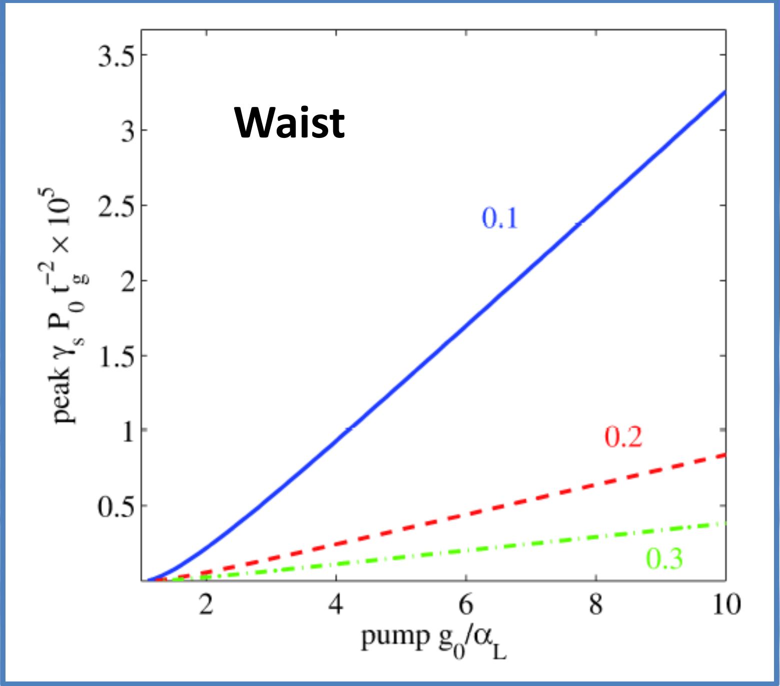
PRL 101, 143901 (2009)

Solution of the Haus/GP equation

E>1 (threshold for RL)







Leonetti and Conti, arXiv: 1005.2082

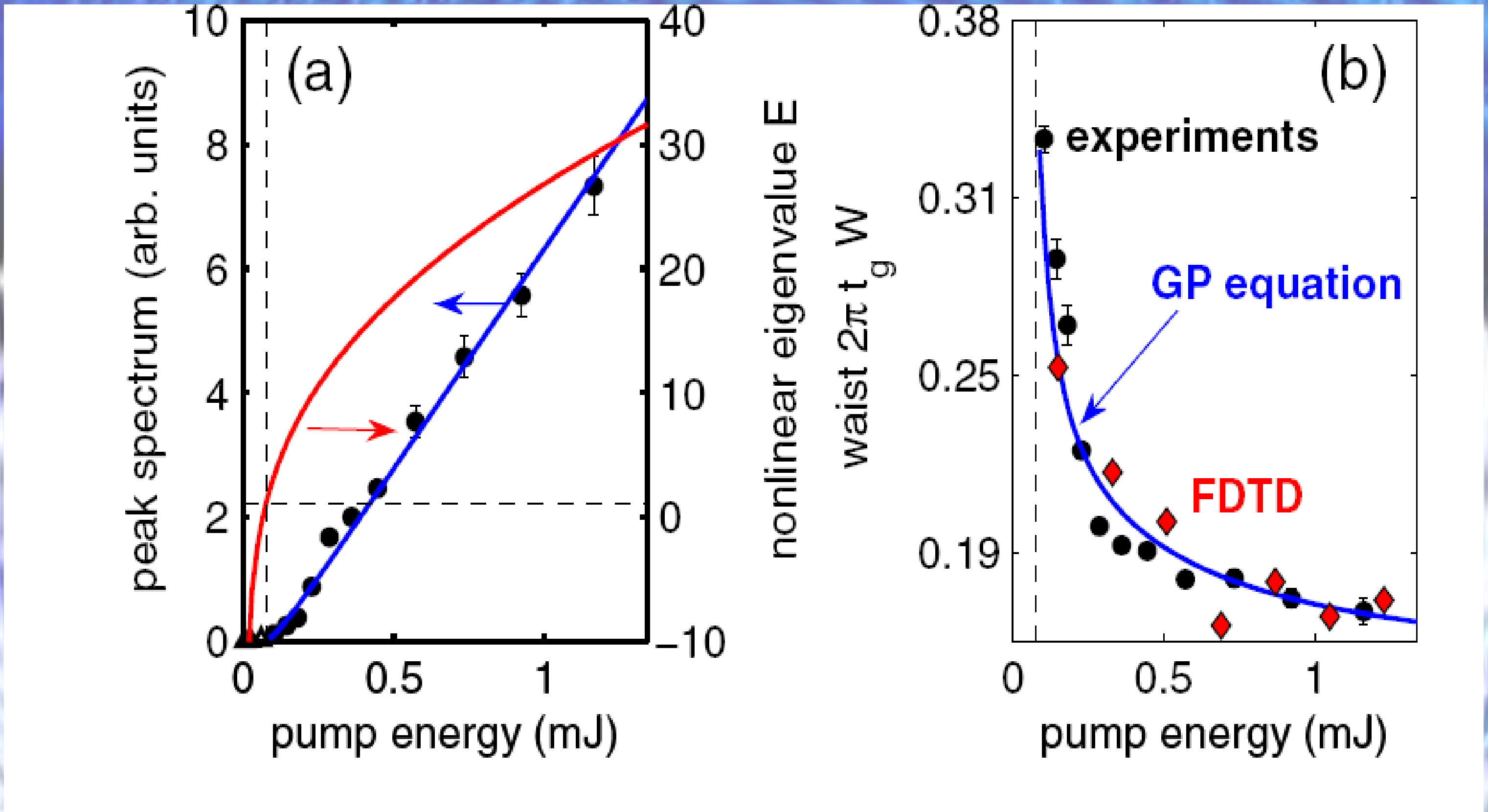
RL Linewidth

$$2\pi t_g W_{th} = \sqrt{\frac{\kappa}{2}} = \sqrt{\frac{t_g}{2t_L}\sqrt{\frac{\alpha_0}{\alpha_L} - 1}}$$

$$\kappa \cong \frac{2\pi t_g}{\lambda}\sqrt{\frac{D}{t_L}}$$

$$W_{th} \cong \sqrt{\frac{1}{4\pi t_g \lambda}\sqrt{\frac{D}{t_L}}}$$

Peak and spectral waist Vs pump



Very similar to a standard laser....

Leonetti and Conti, arXiv: 1005.2082

Conclusions, our results on RL

-Spin glass theory of random lasers

-Haus/Gross Pitaevskii equation for random lasers

-3D FDTD of Anderson localization of light and RL

SPIN GLASS: “The most complex kind of matter”
(Fisher and Hertz)

RANDOM LASER: “The most complex kind of light”

THANKS!
WWW.COMPLEXLIGHT.ORG